An interval-based semantics for degree questions: negative islands and their obviation

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PREVIEW. Fox and Hackl (2006) show that some properly placed modals can obviate negative islands in degree questions, a fact that cannot be handled by earlier accounts of negative islands (Rullmann 1995, Szabolcsi & Zwarts 1993). The generalization offered by F&H states that negative degree questions are unacceptable ((1)), unless negation scopes just over a possibility modal ((2)) or just below a necessity modal ((3)). Their account rests on the assumption that measurement scales are always dense (Universal Density of Measurement – see below). We argue that such a radical step is not warranted: there is no need to resort to the UDM given the independently motivated assumption (Schwarzschild & Wilkinson 2002) that degree predicates are predicates of *intervals* of degrees.

BACKGROUND. F&H's account is based on the following assumptions: (a) The variable bound by the wh-phrase in degree questions ranges over individual degrees: *How tall is John?* = "For what degree d, is it true that John is at least d-tall?". (b) The set all the degrees in a given dimension is always *dense*, i.e. between two distinct degrees there is always a distinct third degree. (c) Dayal (1996): A question presupposes that it has a maximally informative true answer, i.e. a true answer that entails all the true answers. The ungrammaticality of (1) is explained as follows: if John is exactly d tall, then the most informative proposition among the true propositions of the form 'John isn't at least d-tall' would have to be 'John isn't at least d+μ tall', with d+μ the smallest degree above d. But since scales are dense, there can be no such degree, and Dayal (1996)'s presupposition is never met.

PROPOSAL. We maintain (c) but give up (a) and (b). With S & W (2002), we treat degree predicates as predicates of *intervals* of degrees (which is compatible with *discrete* scales, cf. (4)). As a result, the variable bound by a degree operator also ranges over intervals, and a positive degree question like (5)a. receives the representation given in (5)b. Suppose John's height is d. Then any interval I that contains d is such that Jack's height is in I. The most informative true proposition of the form *Jack's height is in I* is obtained by taking I = [d,d] (= {d}). Therefore there is always a maximally informative answer to (5). Consider now negative degree questions. (1) is analyzed as in (6). Suppose Jack's height is 6ft. Then any interval I that doesn't include 6 is such that Jack's height is not in I. The set of all such intervals is the one that includes a) all the intervals contained in [0, 6[(= I_1) and b) all the intervals contained in]6, +∞) (= I_2). Let I_3 be an interval contained in I_2 (e.g. $I_3 = I_2$). Then the (true) proposition that Jack's height is not in I_3 does not entail the proposition that Jack's height is not in I_1. Likewise, for any I_4 included in I_1, the (true) proposition that Jack's height is not in I_4 does not entail that Jack's height is not in I_2. So there is no interval I such that the proposition that Jack's height is not in I entails all the other true propositions of the same form; hence Dayal's presupposition cannot be met, and (1) is predicted to be unacceptable. Note that this account works whether or not the relevant scale is dense. So the unacceptability of *How many children doesn't John have?* is predicted even with the natural assumption that the relevant scale is discrete.

ACCOUNTING FOR MODAL OBVIATION. When a possibility modal intervenes, there are scenarios in which there is a maximally informative true answer**.** For instance, in the case of (2), if the law states that no worker should be exposed to more than a given amount d of radiation, and says nothing more, then the intervals I such that we are not allowed to expose our workers to an amount of radiation included in I are all the intervals that are strictly above d. And the most informative proposition of this form is obtained by taking the largest such interval, i.e. $]d, +\infty)$. All the other cases of obviation uncovered by F & H can be accounted in a similar way, without the UDM. In particular, an existential operator scoping below negation (as in *nobody*) is always predicted to obviate negative islands, irrespective of the precise structure of the relevant domain of quantification. For instance, a few lines of reasoning (given in (9)) show that (8) is predicted to be acceptable and to presuppose the following: for some n, nobody scored n points or more, and for every $m < n$, someone scored exactly m points. So if *ten* is given as an answer, one should understand that the one who scored best scored 9, and that for every number below 9, someone scored that number of points – a prediction that seems to be correct.

REFINING THE PROPOSAL. In order to generate all the existing readings of degree-questions, we need to move to a more sophisticated account in which sets of intervals are not the direct denotation of degree words. Rather, such denotations are derived by means of Schwartzschild's (2004) and Heim's (2006) PI-operator, which turns a predicate of degrees into a predicate of intervals.

(1) *How tall isn't John?
(2) How much radiation are we not allowed to expose our workers to?
(3) How much are you sure that this vessel won't weigh?

(4) a. $[[\text{tall}]] = \lambda I_{<d,t>}$: I is an interval. λx. x's height $\in I$
b. An *interval* is a subset I of a totally ordered set D such that:
$\forall d_1 \forall d_2 (d_1 \in I \;\&\; d_2 \in I) \rightarrow (\forall d_3 (d_3 \in D \;\&\; d_1 < d_3 < d_2) \rightarrow d_3 \in I))$.
Intervals can be defined on any totally ordered domains, **including discrete ones**.

(5) a. How tall is John?
b. For what interval I, Jack's height is in I?

(6) For what interval I, is it true that Jack's height does not belong to I?

(7) a. How much radiation are we not allowed to expose our workers to?
b. For what interval I, we are not allowed to expose our workers to an amount of radiation included in I

(8) How many points did nobody score?
(9) Presupposition of (8): there is an interval I_1 s.t. nobody's score is in I_1 and for every I_2 such that nobody's score is in I_2, the fact that nobody's score is in I_1 entails that nobody's score is in I_2. Let n be the smallest number such that nobody scored n points or more (such a number is sure to exist if the domain of quantification if finite, as we assume). Then clearly nobody's score is in $[n, +\infty)$. Let $J_1 = [n, +\infty)$. Suppose that for some $m < n$, nobody's score is m. Then, for $J_2 = [m, m]$, nobody's score is in J_2. Let J_3 be an interval

that meets (8)'s presupposition, i.e. such that nobody's score is in J_3 and the fact that nobody's score is in J_3 entails all the other true propositions of the same form. Necessarily J_3 includes both J_2 and J_1. Since J_3 has to be an interval and has to include both [m, m] and [n, +∞), it has to include [m, +∞). But then nobody's score is in [m, +∞), and n cannot be the smallest number such that nobody scored n points or more, contrary to what was assumed. So the assumption that, for some m < n, nobody's score is m has to be false. Therefore for every number m below n, someone scored m. And in such a situation, the answer based on the interval [n, +∞) is the most informative answer to (8).

SELECTED REFERENCES

Dayal, V. 1996. *Locality in WH quantification*: Kluwer Academic Publishers Boston.

Fox, D. & M. Hackl, 2006. The universal density of measurement. *Linguistics and Philosophy* 29: 537 586.

Heim, I. 2006. Remarks on comparative clauses as generalized quantifiers. Ms., MIT, *Semantics Archive*: http://semanticsarchive.net/Archive/mJiMDBlN/comparatives%20as%20GQs.pdf

Rullmann, H. 1995. Maximality in the semantics of wh-constructions, University of Massachusetts at Amherst Amherst, Mass.

Schwarzschild, R., and K. Wilkinson. 2002. Quantifiers in Comparatives: A Semantics of Degree Based on Intervals. *Natural Language Semantics* 10:1-41.

Schwarzschild, R., 2004. Scope-splitting in the comparative". Handout from the MIT colloquium, http://www.rci.rutgers.edu/~tapuz/pubs.htm

Szabolcsi, A., and F. Zwarts. 1993. Weak islands and an algebraic semantics for scope taking. *Natural Language Semantics* 1:235-284.